Fermi National Accelerator Laboratory

FERMILAB-Pub-97/207-T

W and Z Transverse Momentum Distributions: Resummation in q_T-space

R.K. Ellis and Siniša Veseli

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

June 1997

Submitted to *Physical Review D*

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Disclaimer

Distribution

FERMILAB-PUB-97/207-T

W and Z transverse momentum distributions: resummation in q_T-space

R.K. Ellis and Siniša Veseli

Theory Group, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510

June 27, 1997

Abstract

We describe an alternative approach to the prediction of W and Z transverse momentum distributions based on an extended version of the DDT formula. The resummation of large logarithms, mandatory at small q_T, is performed in q_T-space, rather than in the impact parameter b. The leading, next-to-leading and next-to-next-to-leading towers of logarithms are identical in the b-space and q_T-space approaches. We argue that these terms are sufficient for W and Z production in the region in which perturbation theory can be trusted. Direct resummation in q_T-space provides a unified description of vector boson transverse momentum distributions valid at both large and small q_T.

1 Introduction

We re-examine the transverse momentum distributions of vector bosons, in view of the large data samples expected at the Tevatron. In $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV we expect about 10^5 W bosons and 10^4 Z bosons, observed through their leptonic decays, per 100 pb^{-1} of accumulated data. These events will be invaluable for QCD studies, as well as for precision measurements of the W mass. In order to exploit these data samples fully the experimenters will require detailed information about the expected rapidity and transverse momentum distributions of the vector bosons and of their decay products.

In QCD a vector boson acquires transverse momentum q_{T} by recoiling against one or more emitted partons [1, 2]. Order by order in perturbation theory we encounter logarithms, $\ln Q^2/q_{\mathrm{T}}^2$, where Q is the mass of the lepton anti-lepton pair resulting from the vector boson decay. These logarithms must be resummed to give an accurate prediction in the low q_{T} region. The original approach to the summation of logarithms at small q_{T} was provided by Dokshitzer, Dykanov and Troyan (DDT) [3] who derived an expression (reproduced here for the case of massive photon production),

$$\frac{d\sigma}{dQ^2 dq_{\mathrm{T}}^2 dy} = \frac{\sigma_0}{Q^2} \sum_q e_q^2 \frac{d}{dq_{\mathrm{T}}^2} \left\{ f_{q/A}(x_{\mathrm{A}}, q_{\mathrm{T}}) f_{\bar{q}/B}(x_{\mathrm{B}}, q_{\mathrm{T}}) \exp[\mathcal{T}_{\mathrm{DDT}}(q_{\mathrm{T}}, Q)] + (q \leftrightarrow \bar{q}) \right\} , \quad (1)$$

where $\mathcal{T}$ is a leading log Sudakov form-factor,[1]

$$\mathcal{T}_{\mathrm{DDT}}(q_{\mathrm{T}}, Q) = -\int_{q_{\mathrm{T}}^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \frac{\alpha_S(\bar{\mu})}{\pi} \frac{4}{3} \Big(\ln \frac{Q^2}{\bar{\mu}^2} - \frac{3}{2} \Big) . \quad (2)$$

The present state of the art in the theoretical description of vector boson production is based on the b-space formalism where b is the impact parameter which is Fourier conjugate to the vector boson transverse momentum. The b-space formalism, which allows the implementation of transverse momentum conservation for the emitted gluons,[2] has

[1]Other notation will be defined in the body of the paper.

[2]See, for example, Ref. [4].

the remarkable consequence that the cross section at $q_{\rm T} = 0$ is calculable for very large Q [5, 6].[3]

Nevertheless, in practice the b-space formalism has certain disadvantages. Since the cross section is given as a Fourier integral in b which extends from 0 to ∞, one cannot make theoretical predictions for *any* $q_{\rm T}$ without having a prescription for dealing with the non-perturbative region of large b. This problem can be solved by introducing an additional non-perturbative form factor (to be determined from experiment), but that also leads to unphysical behaviour of the cross section at large $q_{\rm T}$, where one should recover the ordinary perturbation theory result. These points will be further discussed later in the text.

Clearly, if one could perform the Fourier integral in b analytically and thus obtain an expression for the cross section in $q_{\rm T}$-space, the above problems would be solved. A model for the non-perturbative region would have to be introduced only at the very lowest values of $q_{\rm T}$, and one would have a unified description of vector boson transverse momentum distributions valid at both small and large $q_{\rm T}$.

In this paper we present an approach to resummation in $q_{\rm T}$-space, which is based on an extended version of the DDT formula. The b-space formalism [6]-[14] resums the contributions to the cross section from the following towers of logarithms ($L = \ln Q^2/q_{\rm T}^2$):

$$\begin{aligned} \mathrm{L}: &\quad \frac{1}{q_{\rm T}^2}\alpha_S^j L^{2j-1}\ , \\ \mathrm{NL}: &\quad \frac{1}{q_{\rm T}^2}\alpha_S^j L^{2j-2}\ , \\ \mathrm{NNL}: &\quad \frac{1}{q_{\rm T}^2}\alpha_S^j L^{2j-3}\ , \\ \mathrm{NNNL}: &\quad \frac{1}{q_{\rm T}^2}\alpha_S^j L^{2j-4}\ . \end{aligned} \tag{3}$$

Our extended DDT expression agrees with the b-space results for all but the NNNL series.

[3]Very massive vector bosons are produced at $q_{\rm T} = 0$ in association with semi-hard gluons which have zero net transverse momentum. Unfortunately, in W and Z production Q is too low and this asymptotic regime does not apply.

However, for vector bosons with masses less than M_Z, we find that the NNNL series is numerically unimportant for $q_{\mathrm{T}} > 3$ GeV. Furthermore, a specific choice of coefficients in the q_{T}-space Sudakov form factor allows us to absorb the first term in the NNNL tower of logarithms, and to obtain exact agreement with resummation in b-space to $\mathcal{O}(\alpha_S^2)$.

Based on these results, we argue that the q_{T}-space approach preserves almost all the reliable features of the b-space formalism,[4] and that it also has certain practical advantages:

- We avoid numerical pathologies in the matching, caused by combining results from b-space and q_{T}-space. Although the matching is formally included in the b-space method [7, 10, 12, 13], the cross section is not correctly calculated for $q_{\mathrm{T}} \geq Q/2$. The cross section in this region is the result of a delicate cancellation between the resummed and finite pieces. The slightly different treatment of the two terms is sufficient to upset the cancellation. In contrast, the matching works well in q_{T}-space, leading to a unified description of the q_{T} and y distributions valid for all q_{T}.

- We need to introduce a model only at the very lowest values of q_{T}.

- We have the practical advantage that we avoid both the numerical Fourier transform and multiple evaluations of the structure functions at each value of q_{T}.

A complete explanation of these points will be found later in the paper.

It is important to emphasize here we are not challenging the *theoretical* importance of the b-space formalism, which leads to interesting results, particularly about the production of very massive bosons. Nevertheless, it is our opinion that in practice the extended DDT approach is sufficient for the theoretical description of the W and Z production.

The rest of the paper is organized as follows: in Section 2 we review the b-space resummation. In Section 3 we derive an extended version of the DDT expression, which

[4] The subleading terms in b-space have a profound effect at $q_{\mathrm{T}} = 0$. However, for W and Z production this region is dominated by non-perturbative effects.

forms the basis of our approach. Section 4 contains comparison of the perturbative Sudakov form factors in the $q_{\rm T}$-space and b-space formalisms, and shows that in the region where the latter is reliable it is essentially identical with the former. We also present a prescription for dealing with the non-perturbative region of low $q_{\rm T}$, and compare our results with typical b-space calculations. Our conclusions are given in Section 5, while Appendix A contains the saddle point evaluation of the b-space expression for the cross section at $q_{\rm T} = 0$.

2 Resummation formalism in b-space

The general expression for the resummed differential cross section for vector boson production in hadronic collisions may be written in the form

$$\begin{aligned} \frac{d\sigma(AB \to V(\to l\bar{l}')X)}{dq_T^2\, dQ^2\, dy\, d\cos\theta\, d\phi} &= \frac{1}{2^8 N\pi S}\, \frac{Q^2}{(Q^2 - M_V^2)^2 + M_V^2\Gamma_V^2} \\ &\times \left[Y_r(q_{\rm T}^2, Q^2, y, \theta) + Y_f(q_{\rm T}^2, Q^2, y, \theta, \phi) \right] . \end{aligned} \tag{4}$$

In the above, $N = 3$ is the number of colors, $\sqrt{S}$ is the total hadron-hadron center-of-mass energy, while θ and ϕ refer to the lepton polar and azimuthal angles in the Collins-Soper (CS) frame [15]. The mass and width of the vector boson are denoted by M_V and Γ_V. The functions Y_r and Y_f stand for the resummed and finite parts of the cross section, respectively. As the details of the finite part are not important for the subsequent discussion, we review here only the resummed part, and refer the reader to Ref. [13] for the complete description of $\mathcal{O}(\alpha_S)$ finite part.

The resummed part of the cross section is given as the Fourier integral over the impact parameter b,[5]

$$Y_r(q_{\rm T}^2, Q^2, y, \theta) = \Theta(Q^2 - q_{\rm T}^2)\frac{1}{2\pi}\int_0^\infty db\, b\, J_0(q_{\rm T} b) \sum_{a,b}{}' F_{ab}^{NP}(Q, b, x_{\rm A}, x_{\rm B})$$

[5] The prime on the sum in Eq. (5) indicates that gluons are excluded from the summation.

$$\times \quad W_{ab}(Q, b_*, \theta) f'_{a/A}(x_{\rm A}, \frac{b_0}{b_*}) f'_{b/B}(x_{\rm B}, \frac{b_0}{b_*}) \ , \tag{5}$$

where the variables $x_{\rm A}$ and $x_{\rm B}$ are given in terms of the lepton pair mass Q and rapidity y as

$$x_{\rm A} = \frac{Q}{\sqrt{S}} \exp{(y)} \ , \ \ x_{\rm B} = \frac{Q}{\sqrt{S}} \exp{(-y)} \ . \tag{6}$$

The modified parton structure functions in Eq. (5), f', are related to the $\overline{MS}$ structure functions, f, by a convolution

$$f'_{a/H}(x_{\rm A}, \mu) = \sum_c \int_{x_{\rm A}}^1 \frac{dz}{z} C_{ac} \left(\frac{x_{\rm A}}{z}, \mu \right) f_{c/H} \left(z, \mu \right) \ , \tag{7}$$

where $(a, b \neq g)$ [16]

$$C_{ab}(z, \mu) \ = \ \delta_{ab} \Big\{ \delta(1-z) + \bar{\alpha}_S(\mu) C_F \Big[1 - z + (\frac{\pi^2}{2} - 4) \delta(1-z) \Big] \Big\} \ , \tag{8}$$

$$C_{ag}(z, \mu) \ = \ \bar{\alpha}_S(\mu) T_R \Big[2z(1-z) \Big] \ . \tag{9}$$

Here we have introduced

$$\bar{\alpha}_S(\mu) = \frac{\alpha_S(\mu)}{2\pi} \ , \tag{10}$$

while $C_F = 4/3$ and $T_R = 1/2$ are the usual colour factors.

The function W can be expressed in terms of the Sudakov form factor $\mathcal{S}(b, Q)$ and is given by

$$W_{ab}(Q, b, \theta) = H_{ab}^{(0)}(\theta) \exp{[\mathcal{S}(b, Q)]} \ , \tag{11}$$

where $H^{(0)}$, which includes the angular dependence of the lowest order cross section and coupling factors, is defined in Appendix A of Ref. [13]. The Sudakov form factor itself is given as [6]

$$\mathcal{S}(b, Q) = - \int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \left(\frac{Q^2}{\bar{\mu}^2} \right) A(\bar{\alpha}_S(\bar{\mu})) + B(\bar{\alpha}_S(\bar{\mu})) \right] \ , \tag{12}$$

with $b_0 = 2 \exp(-\gamma_E) \approx 1.1229$. The coefficients A and B are perturbation series in α_S,

$$A(\bar{\alpha}_S) = \sum_{i=1}^{\infty} \bar{\alpha}_S^i A^{(i)} \ , \ \ B(\bar{\alpha}_S) = \sum_{i=1}^{\infty} \bar{\alpha}_S^i B^{(i)} \ . \tag{13}$$

The first two coefficients in the expansion of A and B are known [16, 17]:

$$\begin{aligned}
A^{(1)} &= 2C_F \ , \\
A^{(2)} &= 2C_F\Big(N(\frac{67}{18} - \frac{\pi^2}{6}) - \frac{10}{9}T_R n_f\Big) \ , \\
B^{(1)} &= -3C_F \ , \\
B^{(2)} &= C_F^2\Big(\pi^2 - \frac{3}{4} - 12\zeta(3)\Big) + C_F N\Big(\frac{11}{9}\pi^2 - \frac{193}{12} + 6\zeta(3)\Big) \\
&+ C_F T_R n_f\Big(\frac{17}{3} - \frac{4}{9}\pi^2\Big) \ . \qquad (14)
\end{aligned}$$

One of the main advantages of the b-space resummation formalism is that the simple form for $\mathcal{S}(b,Q)$ as given in Eq. (12), remains valid to all orders in perturbation theory [6]. In addition, as mentioned above, for very large values of the vector boson mass the b-space formulae make definite predictions for the $q_{\mathrm{T}} = 0$ behaviour of the cross section [5, 6].

Unfortunately, the practical implementation of the b-space formulae presents some difficulties. The b-space integral in the Bessel transform in Eq. (5) extends from 0 to ∞, which means that one has to find a way to deal with the non-perturbative region where b is large. That problem is usually circumvented by evaluating W and the parton structure functions at

$$b_* = \frac{b}{\sqrt{1 + (b/b_{\,\mathrm{lim}})^2}} \ , \qquad (15)$$

which never exceeds the cut-off value $b_{\,\mathrm{lim}}$, and also by introducing an additional function F^{NP}, which represents the non-perturbative (large b) part of the Sudakov form factor, to be determined from experiment [6]. This is usually done by assuming a particular functional form for F^{NP} which involves several parameters that can be adjusted in order to give the best possible description of experimental data. The specific choice of the functional form for F^{NP} is a matter of debate [9, 11, 13], but we will not discuss it further here. The point which we would like to emphasize here is that without introducing b_* and F^{NP} one would not be able to make theoretical predictions for *any* value of q_{T}, even in the large q_{T} region where perturbation theory is expected to work well.

Another problem which occurs in the b-space resummation formalism is the transition between the low and the high $q_{\rm T}$ regions. At large $q_{\rm T}$ the resummed part is well represented by the first few terms in its perturbative expansion. When the resummed part Y_r is combined with Y_f one formally recovers the perturbation theory result. However, the cancellation at large $q_{\rm T}$ is quite delicate and is compromised by the non-perturbative function which acts only on Y_r. We illustrate the problem in Figure 1,[6] which compares the $\mathcal{O}(\alpha_S)$ perturbation theory result for $d\sigma/dq_{\rm T}$ in $W^+ + W^-$ production at the Fermilab Tevatron, to the theoretical prediction obtained from the b-space resummation (Eqs. (4) and (5)).[7]

Even though by carefully matching the low and high $q_{\rm T}$ regions one can reduce theoretical errors and produce smoother transverse momentum distributions, matching is still bound to fail eventually, and one is forced to switch to the pure perturbative result at some $q_{\rm T}$ [10]. This procedure inevitably leads to discontinuous $q_{\rm T}$ distributions, which are clearly unphysical.

If one could find a $q_{\rm T}$-space expression for Y_r, both of the above problems would be solved: just as for the conventional perturbation theory, theoretical predictions could be made without any smearing or additional functions, at least for values of $q_{\rm T}$ not too close to zero. Also, since Y_r and Y_f would both be calculated in $q_{\rm T}$-space, the cancellation between the resummed part and subtractions from the finite part would be explicit, and matching of $Y_r + Y_f$ onto the perturbative result at large $q_{\rm T}$ would be manifest. With this motivation we consider the derivation of $q_{\rm T}$-space equivalent of Eq. (5) in the following section.

---

[6]Note that throughout the paper we use the MRSR1 structure functions, with $\alpha_S(M_Z) = 0.113$ [18].

[7]Following Eqs. (22,23) of Ref. [13], instead of b_0/b_* we actually used the exact first order result for the scale at which parton distribution functions are evaluated. This prescription preserves the total integral and reduces to b_0/b_* for large b. Furthermore, it improves the large $q_{\rm T}$ matching between $Y_r + Y_f$ and the perturbation theory.

3 Resummation in q_{T}-space: extended DDT formula

For the sake of simplicity we discuss only the resummed part of the non-singlet (NS) cross section for the process $AB \to \gamma^* X$. The extension to the general process $AB \to V(\to l\bar{l}')X$ is straightforward. In this case Eqs. (4), (5) and (11) can be rewritten in the form

$$
\begin{aligned}
\frac{d\sigma}{dq_T^2\, dQ^2} &= \frac{\sigma_0}{Q^2} \sum_q e_q^2 \int_0^1 dx_{\mathrm{A}} dx_{\mathrm{B}}\, \delta(x_{\mathrm{A}} x_{\mathrm{B}} - \frac{Q^2}{S}) \\
&\times \frac{1}{2} \int_0^\infty db\, b\, J_0(q_{\mathrm{T}} b)\, \exp\left[\mathcal{S}(b,Q)\right] \tilde{f}'_{q/A}(x_{\mathrm{A}}, \frac{b_0}{b})\, \tilde{f}'_{\bar{q}/B}(x_{\mathrm{B}}, \frac{b_0}{b}) \ , \qquad (16)
\end{aligned}
$$

where $\sigma_0 = 4\pi\alpha^2/(9S)$ and $\tilde{f}'_{q/A} = f'_{q/A} - f'_{\bar{q}/A}$, $\tilde{f}'_{\bar{q}/B} = f'_{\bar{q}/B} - f'_{q/B}$ are the higher order NS structure functions. Note that we have removed the non-perturbative function F^{NP} and variable b_* from Eq. (5), so that the above expression represents the pure perturbative result.

From Eq. (16) one can easily obtain the N-th moment of the cross section with respect to $\tau = x_{\mathrm{A}} x_{\mathrm{B}} = Q^2/S$,

$$
\begin{aligned}
\Sigma(N) &= \int d\tau\, \tau^N \frac{Q^2}{\sigma_0} \frac{d\sigma}{dq_T^2\, dQ^2} \\
&= \sum_q e_q^2 \frac{1}{2} \int_0^\infty db\, b\, J_0(q_{\mathrm{T}} b)\, \exp\left[\mathcal{S}(b,Q)\right] \tilde{f}'_{q/A}(N, \frac{b_0}{b})\, \tilde{f}'_{\bar{q}/B}(N, \frac{b_0}{b}) \ . \qquad (17)
\end{aligned}
$$

The N-th moment of the NS higher order structure function satisfies the GLAP equation,[8]

$$
\frac{d}{d \ln \mu^2} f'_{q/H}(N,\mu) = \gamma'_N f'_{q/H}(N,\mu) \ , \qquad (18)
$$

with the solution

$$
\tilde{f}'_{q/H}(N, \frac{b_0}{b}) = \exp\left[- \int_{(b_0/b)^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \gamma'_N(\alpha_S(\bar{\mu})) \right] \tilde{f}'_{q/A}(N, Q) \ . \qquad (19)
$$

Using Eqs. (17,19) we may write

$$
\Sigma(N) = G(N,Q)\, \frac{1}{2} \int_0^\infty db\, b\, J_0(q_{\mathrm{T}} b)\ \exp[\mathcal{U}_N(b,Q)] \ , \qquad (20)
$$

[8] The anomalous dimension γ' differs in a calculable way from the $\overline{MS}$ anomalous dimension.

where $G(N,Q)$ denotes the parton flux,

$$G(N,Q) = \sum_q e_q^2 \; \tilde{f}'_{q/A}(N,Q)\tilde{f}'_{\bar{q}/B}(N,Q) \; , \tag{21}$$

and the exponent $\mathcal{U}$ is given as

$$\begin{aligned} \mathcal{U}_N(b,Q) &= -\int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[A(\bar{\alpha}_S(\bar{\mu})) \ln \frac{Q^2}{\bar{\mu}^2} + B(\bar{\alpha}_S(\bar{\mu})) + 2\gamma'_N(\bar{\alpha}_S(\bar{\mu})) \right] \\ &\equiv \sum_{n=1}^{\infty} \sum_{m=0}^{n+1} \bar{\alpha}_S^n(Q) \ln^m \left(\frac{Q^2 b^2}{b_0^2} \right) \; {}_nD_m \; . \end{aligned} \tag{22}$$

Here ${}_1D_2 = -\frac{1}{2}A^{(1)}$, etc. Inserting Eq. (22) in Eq. (20) we obtain

$$\Sigma(N) = G(N,Q)\, \frac{1}{2} \int_0^\infty db\, b\, J_0(q_{\rm T} b)\, \exp\left[\sum_{n=1}^{\infty} \sum_{m=0}^{n+1} \bar{\alpha}_S^n(Q) \ln^m \left(\frac{Q^2 b^2}{b_0^2} \right) \; {}_nD_m \right] \; . \tag{23}$$

This expression may be integrated by parts using the relationship

$$\frac{d}{dx}\Big[x J_1(x) \Big] = x J_0(x) \; . \tag{24}$$

Because of the rapid damping of the Sudakov factor as $b \to \infty$ we may ignore the boundary terms and obtain

$$\begin{aligned} \Sigma(N) &= -\frac{1}{2q_{\rm T}^2} G(N,Q) \int_0^\infty dx\, J_1(x)\, \frac{d}{dx} \exp\left[\sum_{n=1}^{\infty} \sum_{m=0}^{n+1} \bar{\alpha}_S^n(Q) \ln^m \left(\frac{Q^2 x^2}{q_{\rm T}^2 b_0^2} \right) \; {}_nD_m \right] \; , \\ &\equiv G(N,Q) \int_0^\infty dx\, J_1(x)\, \frac{d}{dq_{\rm T}^2} \exp\left[\sum_{n=1}^{\infty} \sum_{m=0}^{n+1} \bar{\alpha}_S^n(Q) \ln^m \left(\frac{Q^2 x^2}{q_{\rm T}^2 b_0^2} \right) \; {}_nD_m \right] \; . \end{aligned} \tag{25}$$

Eq. (25) already has the structure of the DDT formula. In fact, setting $\ln x/b_0 = 0$ in the integrand we recover exactly the DDT formula, i.e. the exponent has exactly the form of Eq. (22) with b_0/b replaced by $q_{\rm T}$. Because of that we write $\Sigma(N)$ in the form

$$\begin{aligned} \Sigma(N) &= \frac{d}{dq_{\rm T}^2} \left\{ G(N,Q) \int_0^\infty dx\, J_1(x)\, \exp\left[\sum_{n=1}^{\infty} \sum_{m=0}^{n+1} \bar{\alpha}_S^n(Q) \ln^m \left(\frac{Q^2}{q_{\rm T}^2} \right) \; {}_nD_m \right] + R(q_{\rm T}) \right\} \\ &= \frac{d}{dq_{\rm T}^2} \left\{ G(N,Q) \exp[\mathcal{U}_N(\frac{1}{b_0 q_{\rm T}}, Q)] + R(q_{\rm T}) \right\} \\ &\equiv \frac{d}{dq_{\rm T}^2} \left\{ G(N,q_{\rm T}) \exp[\mathcal{S}(\frac{1}{b_0 q_{\rm T}}, Q)] + R(q_{\rm T}) \right\} \; , \end{aligned} \tag{26}$$

where the remainder R is defined as

$$\begin{aligned} R(q_{\rm T}) &= G(N,Q)\int_0^\infty dx\, J_1(x)\Bigg\{\exp\left[\sum_{n=1}^{\infty}\sum_{m=0}^{n+1}\bar\alpha_S^n(Q)\ln^m\left(\frac{Q^2x^2}{q_{\rm T}^2b_0^2}\right)\ {}_nD_m\right] \\ &- \exp\left[\sum_{n=1}^{\infty}\sum_{m=0}^{n+1}\bar\alpha_S^n(Q)\ln^m\left(\frac{Q^2}{q_{\rm T}^2}\right)\ {}_nD_m\right]\Bigg\} . \end{aligned} \tag{27}$$

Using[9]

$$\int_0^\infty dx\, J_1(x)\left\{1,\ln\frac{x}{b_0},\ln^2\frac{x}{b_0},\ln^3\frac{x}{b_0},\dots\right\} = \left\{1,0,0,-\frac{1}{2}\zeta(3),\dots\right\} , \tag{28}$$

we can evaluate $R(q_{\rm T})$ as a power series in α_S. We find that the remainder contributes to the NNNL tower of terms, three logarithms down from the leading terms ($L=\ln Q^2/q_{\rm T}^2$),

$$R(q_{\rm T}) = -G(N,Q)\Bigg\{\zeta(3)\sum_{j=2}^{\infty} r_j({}_1D_2)^j\bar\alpha_S^j(Q)L^{2j-3} + O(\bar\alpha_S^jL^{2j-4})\Bigg\} , \tag{29}$$

with

$$\left\{r_2,r_3,r_4,r_5,r_6,r_7,\dots\right\} = \left\{8,\frac{40}{3},\frac{28}{3},4,\frac{11}{9},\frac{13}{45},\dots\right\} . \tag{30}$$

Starting from the b-space expression we have demonstrated an extended DDT formula,

$$\begin{aligned} \frac{d\sigma}{dq_T^2\,dQ^2} &= \frac{\sigma_0}{Q^2}\sum_q e_q^2\int_0^1 dx_{\rm A}dx_{\rm B}\,\delta(x_{\rm A}x_{\rm B}-\frac{Q^2}{S}) \\ &\times\ \frac{d}{dq_{\rm T}^2}\Bigg\{\tilde f'_{q/A}(x_{\rm A},q_{\rm T})\,\tilde f'_{\bar q/B}(x_{\rm B},q_{\rm T})\,\exp\left[\mathcal{T}(q_{\rm T},Q)\right] + O(\bar\alpha_S^jL^{2j-3})\Bigg\} , \end{aligned} \tag{31}$$

which holds if we drop NNNL terms. In the above expression the $q_{\rm T}$-space Sudakov form factor is given by

$$\mathcal{T}(q_{\rm T},Q) = -\int_{q_{\rm T}^2}^{Q^2}\frac{d\bar\mu^2}{\bar\mu^2}\left[\tilde A(\bar\alpha_S(\bar\mu))\ln\frac{Q^2}{\bar\mu^2}+\tilde B(\bar\alpha_S(\bar\mu))\right] , \tag{32}$$

where the $q_{\rm T}$-space coefficients $\tilde A$ and $\tilde B$ are defined in a similar way as their b-space counterparts, i.e.

$$\tilde A(\alpha_S) = \sum_{i=1}^{\infty}\bar\alpha_S^i\tilde A^{(i)} ,\ \tilde B(\alpha_S) = \sum_{i=1}^{\infty}\bar\alpha_S^i\tilde B^{(i)} . \tag{33}$$

[9]See, for example, Ref. [4].

The first two coefficients in $\tilde{A}$ and $\tilde{B}$ would be exactly the same as corresponding b-space coefficients if we drop NNNL terms. However, by making the particular choice of

$$\begin{aligned}
\tilde{A}^{(1)} &= A^{(1)} , \\
\tilde{A}^{(2)} &= A^{(2)} , \\
\tilde{B}^{(1)} &= B^{(1)} , \\
\tilde{B}^{(2)} &= B^{(2)} + 2(A^{(1)})^2 \zeta(3) , \qquad (34)
\end{aligned}$$

we absorb the first term in the NNNL tower of logarithms. In this way Eq. (34) imposes exact agreement between the b-space and q_{T}-space formalisms at order α_S^2.

As we pointed out at the beginning of this section, the extension of the NS cross section for $AB \to \gamma^* X$ to the general case of $AB \to V(\to l\bar{l}')X$ which includes the decay presents no difficulties, so that our q_{T}-space equivalent of Eq. (5) is given in the extended DDT form as

$$\begin{aligned}
\tilde{Y}_r(q_{\mathrm{T}}^2, Q^2, y, \theta) &= \Theta(Q^2 - q_{\mathrm{T}}^2) \frac{1}{\pi} \\
&\times \sum_{a,b}{}' H_{ab}^{(0)}(\theta) \frac{d}{dq_{\mathrm{T}}^2} \left[f'_{a/A}(x_{\mathrm{A}}, q_{\mathrm{T}}) \, f'_{b/B}(x_{\mathrm{B}}, q_{\mathrm{T}}) \, \exp\left[\mathcal{T}(q_{\mathrm{T}}, Q)\right] \right] , \qquad (35)
\end{aligned}$$

with $\mathcal{T}$ given in Eq. (32) in terms of coefficients of Eqs. (33,34). The above equation is the central result of this paper. It is still ill-defined in the small q_{T} region, which reflects the fact that the problem is not entirely determined by perturbation theory and requires non-perturbative input. We will discuss our model for the non-perturbative region later in the following section.

4 Results

4.1 Form factors

Before presenting our results for W and Z production we compare the form factors calculated using the b-space and q_{T}-space formulae, for values of Q which are presently of interest. In practice this means $Q \leq M_Z$. The comparison of the form factors will allow us to make an estimate of the practical numerical importance of transverse momentum conservation, i.e. of the subleading terms which are not present in the $q_{\rm T}$-space formalism. To simplify the comparison we will consider the effects of the Sudakov form factor alone. We will therefore ignore the influence of modified parton distribution functions on the $q_{\rm T}$ dependence. For the purpose of illustration we take $Q = M_Z$ and $\alpha_S(M_Z) = 0.113$.

We define the b-space form factor as

$$\begin{aligned} F^{(b)}(q_{\rm T}) &= \frac{Q^2}{4\pi} \int d^2b \; \exp(ib \cdot q_{\rm T}) \; \exp[\mathcal{S}(b_*, Q)] \; F^{NP}(Q, b, x_{\rm A}, x_{\rm B}) \\ &= \frac{Q^2}{2} \int_0^\infty db \, b \, J_0(bq_{\rm T}) \; \exp[\mathcal{S}(b_*, Q)] \; F^{NP}(Q, b, x_{\rm A}, x_{\rm B}) \; . \end{aligned} \tag{36}$$

Note that F^{NP} and b_* have to be introduced in the above expression as a prescription for dealing with the non-perturbative region of large b. A specific choice of the non-perturbative function should make a difference only in the region of low $q_{\rm T}$. In order to show that, in Figure 2 we present form factors evaluated with F^{NP} taken from Ref. [11] (LY), and with an effective gaussian as used in Ref. [13] (ERV, $g = 3.0$ GeV^2). For LY form factor we take $x_{\rm A} = x_{\rm B} = M_Z/\sqrt{S}$ for $\sqrt{S} = 1.8$ TeV. As expected, at large $q_{\rm T}$ the form factors resulting from the two choices of F^{NP} agree. For small $q_{\rm T}$ we find that results for $F^{(b)}(q_{\rm T})$ tend to a different finite intercept controlled by the non-perturbative function.

The above b-space expression for the form factor should be compared to its $q_{\rm T}$-space counterpart,

$$F^{(q_{\rm T})}(q_{\rm T}) = Q^2 \frac{d}{dq_{\rm T}^2} \exp[\mathcal{T}(q_{\rm T}, Q)] \; , \tag{37}$$

and also to the $\mathcal{O}(\alpha_S^2)$ perturbation theory result,

$$F^{(p)}(q_{\rm T}) = \frac{Q^2}{q_{\rm T}^2} \sum_{n=1}^{2} \sum_{m=0}^{2n-1} \bar{\alpha}_S^n \ln^m \frac{Q^2}{q_{\rm T}^2} \; {}_nC_m \; , \tag{38}$$

with ${}_nC_m$ given in terms of the $q_{\rm T}$-space coefficients as

$${}_1C_1 = \tilde{A}^{(1)} \; ,$$

$$
\begin{aligned}
{}_1C_0 &= \tilde{B}^{(1)} , \\
{}_2C_3 &= -\frac{1}{2}\left(\tilde{A}^{(1)}\right)^2 , \\
{}_2C_2 &= \tilde{A}^{(1)}(\beta_0 - \frac{3}{2}\tilde{B}^{(1)}) , \\
{}_2C_1 &= \tilde{A}^{(2)} + \tilde{B}^{(1)}(\beta_0 - \tilde{B}^{(1)}) , \\
{}_2C_0 &= \tilde{B}^{(2)} .
\end{aligned}
\tag{39}
$$

As one can see from Figure 3, in the region where one can trust perturbation theory ($q_{\mathrm{T}} \geq 3$ GeV), our q_{T}-space result of Eq. (37) agrees well with the b-space form factor (obtained with ERV non-perturbative function). Further, it is clear that resummation is needed in the region where $F^{(b)}$ and $F^{(q_{\mathrm{T}})}$ differ significantly from the perturbative result.

It is also interesting to investigate the size of the NNNL effects. In Figure 4 we show the q_{T}-space form factor $F^{(q_{\mathrm{T}})}(q_{\mathrm{T}})$ calculated using coefficients given in Eq. (34), and also the one calculated with $\tilde{B}^{(2)}$ replaced by $B^{(2)}$. As one can see, the change is never more than a few percent for $q_{\mathrm{T}} > 3$ GeV.

We therefore conclude that the b-space and q_{T}-space formula are substantially identical, despite the neglect of NNNL terms in the latter. The differences between them are smaller than the differences introduced in the b-space formalism by the use of different non-perturbative functions. The above conclusion holds for the particular case of the vector boson production with $Q \leq M_Z$.

4.2 Extension to the non-perturbative region

As we have already pointed out, there are two main advantages of the q_{T}-space approach over the b-space formalism: first, outside of the non-perturbative region one can make theoretical predictions based on perturbation theory alone with soft gluon resummation effects included. In Figures 5 and 6 we show predictions of Eq. (35) for $W^+ + W^-$ and Z production at Fermilab Tevatron. It is clear that these predictions are quite close to typical b-space results. Second, matching of the resummation formalism onto pure perturbation theory for large $q_{\rm T}$ is explicit, and hence there is no need for somewhat unnatural switching from one type of theoretical description to another. Since our calculation contains the $\mathcal{O}(\alpha_S)$ finite part and the $\mathcal{O}(\alpha_S^2)$ Sudakov form factor, there still may be some residual unmatched higher order effects present in $d\sigma/dq_{\rm T}$ in the large $q_{\rm T}$ region, where the cancellation of the resummed part and subtractions from the finite part is quite delicate. However, these effects are expected to be small, and should be even less important after the inclusion of the second order calculation of Y_f. The $q_{\rm T}$-space matching is illustrated in Figure 7 for $W^+ + W^-$ production at Tevatron, and should be compared to the b-space result shown in Figure 1. Note that less than 2% of the total cross section lies above $q_{\rm T} = 50$ GeV, so the overall importance of the portion of the cross section shown in Fig. 7 is quite small.

Up to now we have discussed only the $q_{\rm T}$-space predictions in the perturbative region, i.e. for $q_{\rm T} \geq 2 - 3$ GeV. Still, in order to compare theoretical predictions to experiment one has to find a way of dealing with the non-perturbative region ($q_{\rm T} \to 0$), where Eq. (35) is ill-defined. The form of $\tilde{Y}_r$ suggests that we make the following replacement in Eq. (35):

$$\begin{aligned} f'_{a/A}(x_{\rm A}, q_{\rm T})\, f'_{b/B}(x_{\rm B}, q_{\rm T}) \, \exp\left[\mathcal{T}(q_{\rm T}, Q)\right] \;\longrightarrow & \\ f'_{a/A}(x_{\rm A}, q_{\rm T*})\, f'_{b/B}(x_{\rm B}, q_{\rm T*}) \, \exp\left[\mathcal{T}(q_{\rm T*}, Q)\right] \tilde{F}^{NP}(q_{\rm T}) \; . & \qquad (40) \end{aligned}$$

Here, $q_{\rm T*}$ is the effective transverse momentum and $\tilde{F}^{NP}$ is the $q_{\rm T}$-space non-perturbative part of the form factor. Since the above replacement should affect only the region of small $q_{\rm T}$, we define $q_{\rm T*}$ as

$$q_{\rm T*}^2 = q_{\rm T}^2 + q_{\rm Tlim}^2 \exp\left[-\frac{q_{\rm T}^2}{q_{\rm Tlim}^2}\right] , \qquad (41)$$

which never goes below the limiting value $q_{\rm Tlim}$, and also approaches $q_{\rm T}$ as $q_{\rm T}$ becomes much larger than $q_{\rm Tlim}$. For $\tilde{F}^{NP}$ we require that

$$\begin{aligned} \tilde{F}^{NP}(q_{\rm T}) &\;\to\; 0 \qquad (\text{for } q_{\rm T} \to 0) \; , \\ \tilde{F}^{NP}(q_{\rm T}) &\;\to\; 1 \qquad (\text{for } q_{\rm T} \to Q) \; , \\ \frac{d}{dq_{\rm T}^2}\tilde{F}^{NP}(q_{\rm T}) &\;\to\; const. \quad (\text{for } q_{\rm T} \to 0) \; . \qquad (42) \end{aligned}$$

The first two properties ensure that the integral of $\tilde{Y}_r$ over q_{T}^2 gives the result

$$\int_0^{Q^2} dq_{\mathrm{T}}^2 \; \tilde{Y}_r(q_{\mathrm{T}}^2, Q^2, y, \theta) = \frac{1}{\pi} \sum_{a,b}{}' H_{ab}^{(0)}(\theta) f'_{a/A}(x_{\mathrm{A}}, Q) f'_{b/B}(x_{\mathrm{B}}, Q) \; , \tag{43}$$

which is required to reproduce the exact $O(\alpha_S)$ total cross section after integration over q_{T}, as explained in Ref. [13]. The third condition is motivated by the analytic b-space results for $d\sigma/dq_{\mathrm{T}}^2$ in the limit where $q_{\mathrm{T}} \to 0$ [5] (see Appendix A).

A simple choice for $\tilde{F}^{NP}$ which satisfies all of the above requirements is

$$\tilde{F}^{NP}(q_{\mathrm{T}}) = 1 - \exp\left[-\tilde{a}\, q_{\mathrm{T}}^2\right] . \tag{44}$$

At $q_{\mathrm{T}} = 0$ this yields

$$\begin{aligned} \frac{d\sigma}{dq_{\mathrm{T}}^2} \;&\propto\; \tilde{a} \sum_{a,b}{}' H_{ab}^{(0)}(\theta) \Big[f'_{a/A}(x_{\mathrm{A}}, q_{\mathrm{Tlim}})\, f'_{b/B}(x_{\mathrm{B}}, q_{\mathrm{Tlim}}) \, \exp\left[\mathcal{T}(q_{\mathrm{Tlim}}, Q)\right] \Big] \; , \\ \frac{d}{dq_{\mathrm{T}}^2}\Big(\frac{d\sigma}{dq_{\mathrm{T}}^2}\Big) \;&\propto\; -\tilde{a}^2 \sum_{a,b}{}' H_{ab}^{(0)}(\theta) \Big[f'_{a/A}(x_{\mathrm{A}}, q_{\mathrm{Tlim}})\, f'_{b/B}(x_{\mathrm{B}}, q_{\mathrm{Tlim}}) \, \exp\left[\mathcal{T}(q_{\mathrm{Tlim}}, Q)\right] \Big] \; . \end{aligned} \tag{45}$$

Therefore, $\tilde{a}$ and q_{Tlim} control the intercept and the first derivative of $d\sigma/dq_{\mathrm{T}}^2$ at $q_{\mathrm{T}} = 0$. The effects of changing these non-perturbative parameters are illustrated in Figures 8 and 9, for $W^+ + W^-$ production at Fermilab Tevatron. In Figure 8 we compare typical b-space results for the $d\sigma/dq_{\mathrm{T}}^2$ distribution, to the q_{T}-space predictions with several different values of $\tilde{a}$ (q_{Tlim} was fixed to 4.0 GeV). In Figure 9 we plot our $d\sigma/dq_{\mathrm{T}}$ results obtained with $\tilde{a}$ fixed to 0.10 GeV^{-2}, and for several different choices of q_{Tlim}. These results show how varying q_{Tlim} modifies the width and shifts the peak of the $d\sigma/dq_{\mathrm{T}}$ distribution.

From Figures 8 and 9 it is also clear that $\tilde{a}$ and q_{Tlim} affect only the low q_{T} region, while for $q_{\mathrm{T}} \geq 10$ GeV we again obtain the extended DDT result of Eq. (35). Because of that, determination of these parameters from the experimental data should not be too difficult.

4.3 Overall smearing

Introduction of $\tilde{a}$ and q_{Tlim} allowed us to extend the validity of Eq. (35) beyond the perturbative region in q_{T}. However, this may not be enough for a good description of experimental data, and one may need additional degrees of freedom for modelling the low q_{T} region.[10] This can be achieved by choosing more complicated functional forms for $q_{\mathrm{T}*}$ and $\tilde{F}^{NP}$ than the ones we suggested in Eqs. (41, 44), or by imposing an overall smearing on the theoretical transverse momentum distributions. Here we briefly discuss the later possibility.

Suppressing irrelevant variables, the smeared cross section is given in terms of

$$\tilde{Y}_i(q_{\mathrm{T}}^2) = \int d^2k_{\mathrm{T}} f(|\mathbf{k}_{\mathrm{T}} - \mathbf{q}_{\mathrm{T}}|)\, \tilde{Y}_i(k_{\mathrm{T}}^2) \ , \tag{46}$$

where $\tilde{Y}_i$ stands for either resummed or finite part in q_{T}-space, and f is the smearing function. For the sake of simplicity we take a gaussian,

$$f(k_{\mathrm{T}}) = \frac{\tilde{g}}{\pi} \exp(-\tilde{g}\, k_{\mathrm{T}}^2) \ , \tag{47}$$

with $\tilde{g}$ being an additional non-perturbative parameter. The above choice is convenient since the azimuthal integration can be done analytically. This leads to the final expression for the smeared $\tilde{Y}_i$,

$$\tilde{Y}_i(q_{\mathrm{T}}^2) = \tilde{g} \int dk_{\mathrm{T}}^2 \ \exp\Big[- \tilde{g}(q_{\mathrm{T}}^2 + k_{\mathrm{T}}^2)\Big]\, I_0(2\tilde{g} q_{\mathrm{T}} k_{\mathrm{T}})\, \tilde{Y}_i(k_{\mathrm{T}}^2) \ . \tag{48}$$

Note that both the resummed and the finite part of the cross section can be smeared together, and therefore the smearing procedure should not affect matching onto the pure perturbative result at large q_{T}. The effects of an overall smearing for low q_{T} are illustrated in Figure 10 for $W^+ + W^-$ production at Tevatron.

[10] We remind the reader that some choices of the non-perturbative function in the b-space formalism involve 4-6 different parameters.

5 Conclusions

In this paper we have outlined an approach to the calculation of the transverse momentum distributions of W and Z bosons using an extension of the DDT formula which works directly in $q_{\rm T}$-space. Our formalism agrees with b-space for all calculated logarithms except the NNNL series. This is a pragmatic approach which uses the available theoretical in an efficient way. For $q_{\rm T}$ above about 3 GeV the cross section is essentially determined by perturbative QCD. In the region $q_{\rm T} \leq 3$ GeV the cross section is determined by a model, the form of which is motivated by the analytic results from the b-space approach. Just as in the b-space approach, the details of the model are to be fixed by comparison with experiment. The numerical program incorporating our results describes all kinematic regions.

An obvious shortcoming of this paper is the failure to include the results of the order α_S^2 calculations [17, 19, 20] (generalized to include the decay of the vector boson [21, 22]) in the finite part of the cross section. In the $q_{\rm T}$-space formalism these should be relatively straightforward to include. After inclusion of these effects we will have a full description of vector boson production valid in all kinematic regions, with a minimum of model dependence.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76CH03000.

A Analytic behaviour at $q_{\rm T} = 0$

The result of Parisi and Petronzio [5] for the intercept at $q_{\rm T} = 0$ can be obtained by saddle point evaluation of Eq. (36),

$$F^{(b)}(0) = \frac{Q^2}{4} \int db^2 \exp[\mathcal{S}(b, Q)] \ . \tag{49}$$

In writing the above equation we have assumed that the saddle point value of b is small so that $b_* = b$ and $F^{NP}(b) = 1$. Introducing the variable $x = \ln b^2$ we have that

$$F^{(b)}(0) = \frac{Q^2}{4} \int dx \exp[-h(x)] \ , \tag{50}$$

where

$$h(x) = -[x + \mathcal{S}(\exp(x/2), Q)] \ . \tag{51}$$

The saddle point result for $F^{(b)}(0)$ is then given by

$$F^{(b)}(0) = \frac{Q^2}{4} \sqrt{\frac{2\pi}{h''(x_{\rm SP})}} \exp[-h(x_{\rm SP})] \ , \tag{52}$$

where $x_{\rm SP} = \ln b_{\rm SP}^2$ is defined by the condition

$$h'(x_{\rm SP}) = 0 \ . \tag{53}$$

On the assumption that the structure functions are slowly varying functions of the scale, the resummed part at $q_{\rm T} = 0$ becomes

$$Y_r(0, Q^2, y, \theta) = \frac{b_{\rm SP}^2}{4\pi} \sqrt{\frac{2\pi}{-\mathcal{S}''(b_{\rm SP}, Q)}} \sum_{a,b}{}' W_{ab}(Q, b_{\rm SP}, \theta) f'_{a/A}(x_{\rm A}, \frac{b_0}{b_{\rm SP}}) f'_{b/B}(x_{\rm B}, \frac{b_0}{b_{\rm SP}}) \ , \tag{54}$$

where

$$\mathcal{S}''(b, Q) = \frac{d^2\mathcal{S}(b, Q)}{d(\ln b^2)^2} \ . \tag{55}$$

By retaining only the leading term (A_1) in the Sudakov form factor we can obtain an approximate analytic solution. We assume that the running coupling satisfies the equation ($\beta_0 = (33 - 2n_f)/6$)

$$\alpha_S(\mu) = \frac{2\pi}{\beta_0} \frac{1}{\ln \mu^2/\Lambda^2} \ , \tag{56}$$

and set $C = 2C_F/\beta_0$. The saddle point of the integral is then given by

$$\frac{1}{b_{\rm SP}} = \frac{\Lambda}{b_0} \Big(\frac{Q}{\Lambda}\Big)^{\frac{C}{C+1}} \ . \tag{57}$$

Using Eqs. (54,57) we obtain the final result for the resummed part of the cross section ($L = \ln Q^2/\Lambda^2$),

$$Y_r(0, Q^2, y, \theta) \approx \frac{b_0^2}{4\pi\Lambda^2} \frac{\sqrt{2\pi C L}}{(C+1)} \Big(\frac{\Lambda^2}{Q^2}\Big)^\eta {\sum_{a,b}}' H_{ab}^{(0)}(\theta) f'_{a/A}(x_{\rm A}, \frac{b_0}{b_{\rm SP}}) f'_{b/B}(x_{\rm B}, \frac{b_0}{b_{\rm SP}}) \ , \tag{58}$$

with

$$\eta = C \ln \frac{C+1}{C} \ . \tag{59}$$

The Y_r has a finite intercept at $q_{\rm T} = 0$ which shrinks with Q. For $n_f = 3\,(4)$ we have that $\eta = 0.586\,(0.602)$.

References

[1] H. Fritzsch and P. Minkowski, *Phys. Lett.* **B73**, 80 (1978);
G. Altarelli, G. Parisi and R. Petronzio, *Phys. Lett.* **B76**, 351 (1978);
K. Kajantie and R. Raitio, *Nucl. Phys.* **B139**, 72 (1978);
F. Halzen and D.M. Scott, *Phys. Rev.* **D18**, 3379 (1978).

[2] For a review see, R.K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics*, Cambridge University Press, (1996).

[3] Yu.L. Dokshitzer, D.I. Dyakonov and S.I. Troyan, *Phys. Rep.* **58**, 269 (1980).

[4] S.D. Ellis, N. Fleishon and W.J. Stirling, *Phys. Rev.* **24**, 1386 (1981).

[5] G. Parisi and R. Petronzio, *Nucl. Phys.* **B154**, 427 (1979).

[6] J. Collins, D. Soper and G. Sterman, *Nucl. Phys.* **B250**, 199 (1985).

[7] G. Altarelli, R.K. Ellis, M. Greco, G. Martinelli, *Nucl. Phys.* **B246**, 12 (1984).

[8] J.C. Collins and D. Soper, *Nucl. Phys.* **B193**, 381 (1981); Erratum *Nucl. Phys.* **B213**, 545 (1983); *Nucl. Phys.* **B197**, 446 (1982).

[9] C. Davies, B.R. Webber and W.J. Stirling, *Nucl. Phys.* **B256**, 413 (1985).

[10] P.B. Arnold and R.P. Kauffman, *Nucl. Phys.* **B349**, 381 (1991).

[11] G.A. Ladinsky and C.-P. Yuan, *Phys. Rev.* **D50**, 4239 (1994).

[12] C. Balázs, J. Qiu, and C.–P. Yuan, *Phys. Lett.* **B355**, 548 (1995).

[13] R.K. Ellis, D.A. Ross and S. Veseli, Fermilab preprint FERMILAB-PUB-97/082-T, hep-ph/9704239 (*Nucl. Phys.* **B**, in press).

[14] C. Balázs and C.–P. Yuan, MSUHEP–70402, hep-ph/9704258.

[15] J. Collins and D. Soper, *Phys. Rev.* **D16**, 2219 (1977).

[16] C. Davies and W.J. Stirling, *Nucl. Phys.* **B244**, 337 (1984);
C.T.H. Davies, University of Cambridge Ph.D. thesis, (1984).

[17] R.K. Ellis, G. Martinelli and R. Petronzio, *Nucl. Phys.* **B211**, 106 (1983).

[18] A.D. Martin, R.G. Roberts and W.J. Stirling, *Phys. Lett.* **B387**, 419 (1996).

[19] P. Arnold and M.H. Reno, *Nucl. Phys.* **B319**, 37 (1989), Erratum *Nucl. Phys.* **B330**, 284 (1990);
P. Arnold, R.K. Ellis and M.H. Reno, *Phys. Rev.* **D40**, 912 (1989).

[20] R. Gonsalves, J, Pawlowski and C-F Wai, *Phys. Rev.* **D40**, 2245 (1989).

[21] E. Mirkes, *Nucl. Phys.* **B387**, 3 (1992);
E. Mirkes and J. Ohnemus, *Phys. Rev.* **D51**, 4891 (1995).

[22] W. Giele, E. Glover, D.A. Kosower, *Nucl. Phys.* **B403**, 633 (1993),
Phys. Lett. **B309**, 205 (1993).

[23] F. Abe *et al.*, *Phys. Rev. Lett.* **66**, 2951 (1991).

[24] F. Abe *et al.*, *Phys. Rev. Lett.* **67**, 2937 (1991).

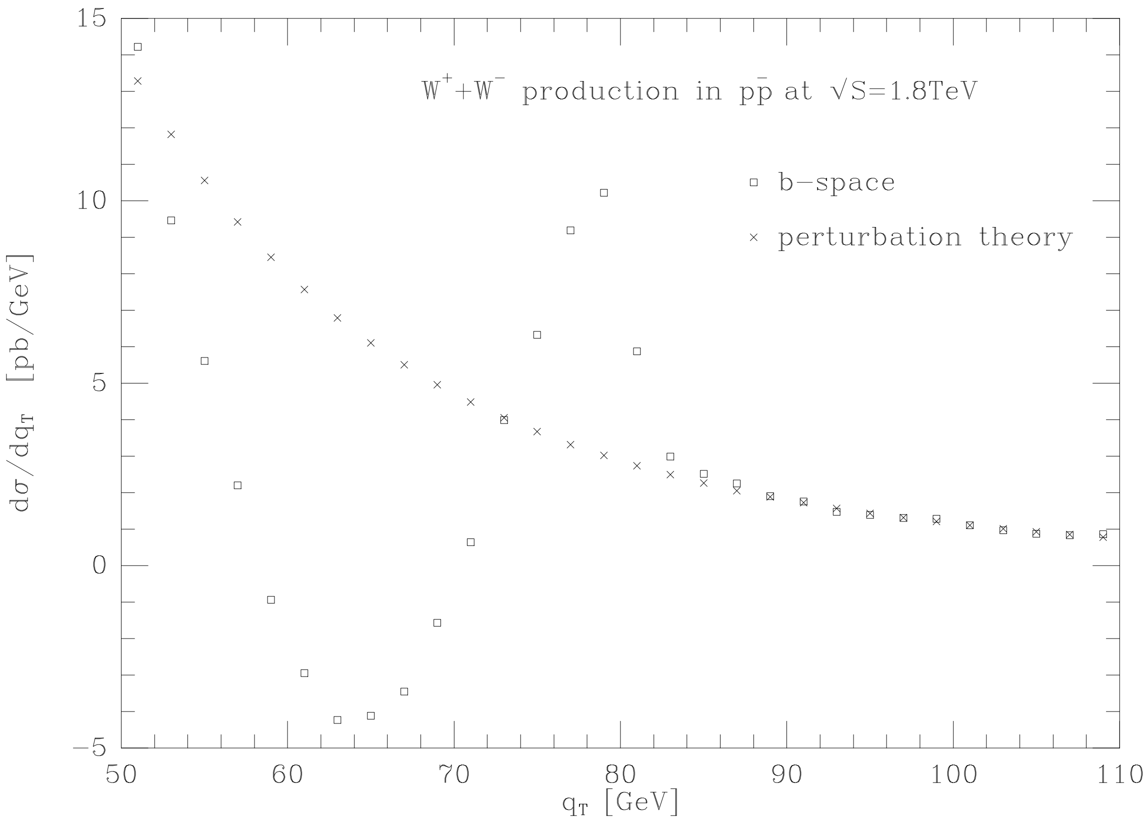

Figure 1: Comparison of the b-space $d\sigma/dq_{\rm T}$ distribution for $W^+ + W^-$ production at $\sqrt{S} = 1.8$ TeV with $\mathcal{O}(\alpha_S)$ perturbative calculation. The resummation results were obtained with pure gaussian ($g = 3.0$ GeV$^2, b_{\rm lim} = 0.5$ GeV^{-1}) form of F^{NP}. We assumed $BR(W \to e\nu) = 0.111$.

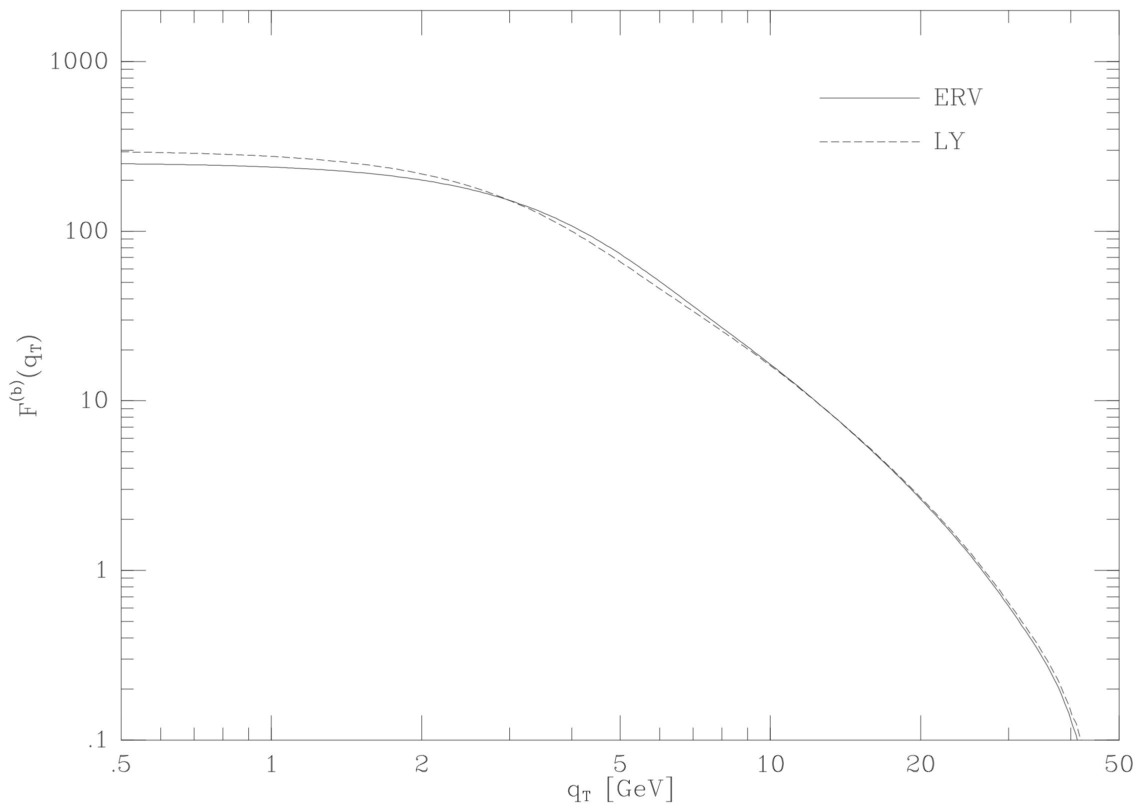

Figure 2: $F^{(b)}(q_{\rm T})$ for the two different choices of the non-perturbative function.

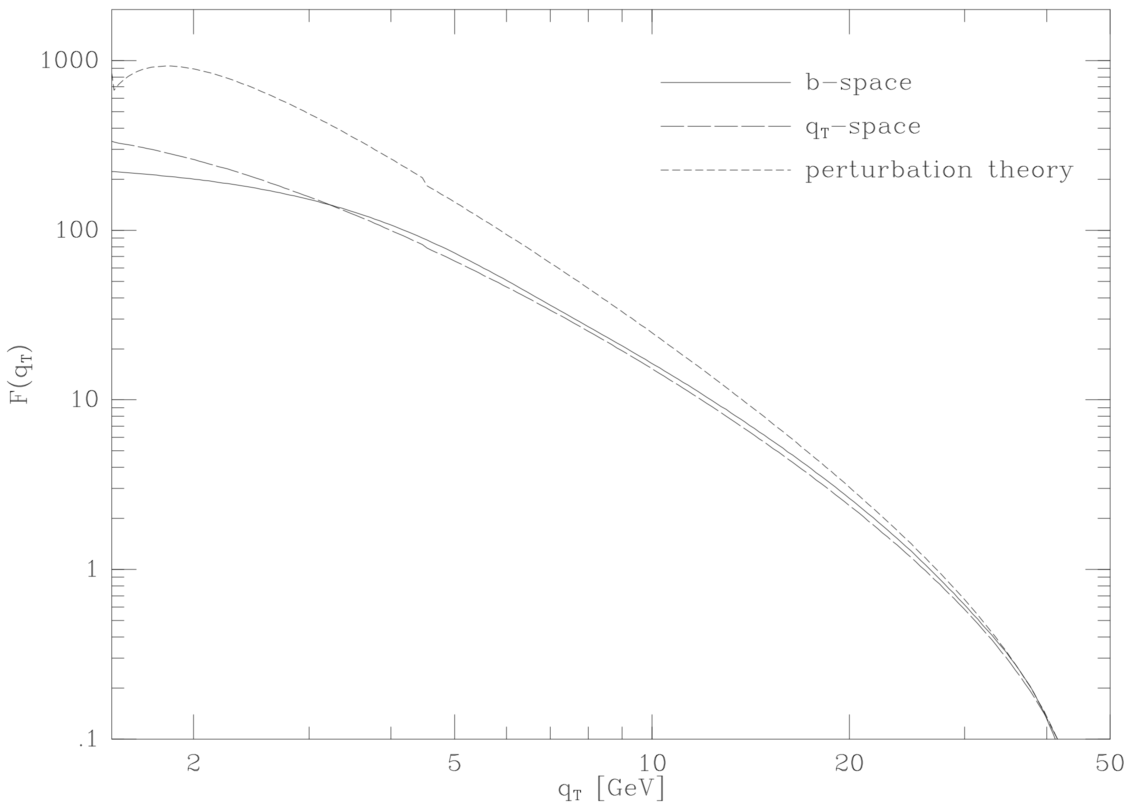

Figure 3: Form factors $F^{(b)}$, $F^{(q_{\rm T})}$ and $F^{(p)}$. The b-space results were obtained with an effective gaussian form of F^{NP} ($g = 3.0$ GeV$^2, b_{\rm lim} = 0.5$ GeV^{-1}).

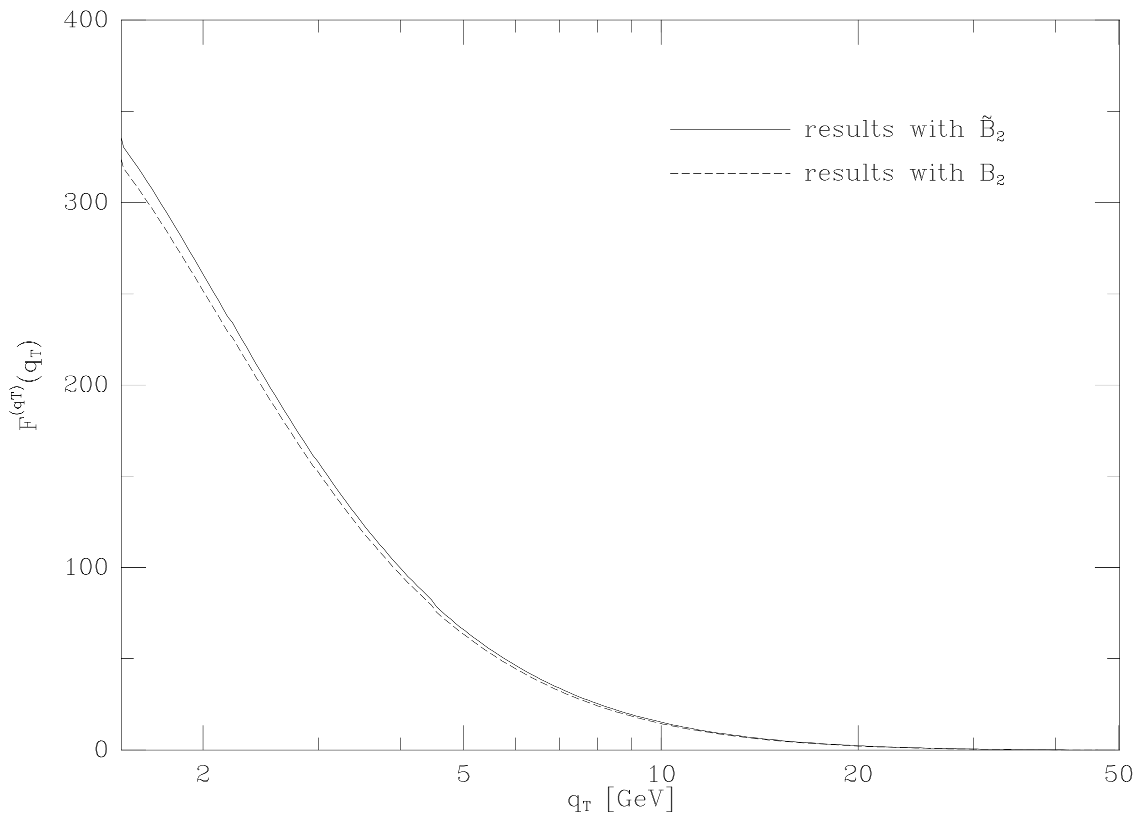

Figure 4: The $q_{\rm T}$-space form factor $F^{(q_{\rm T})}$ calculated with $B^{(2)}$ and $\tilde{B}^{(2)}$.

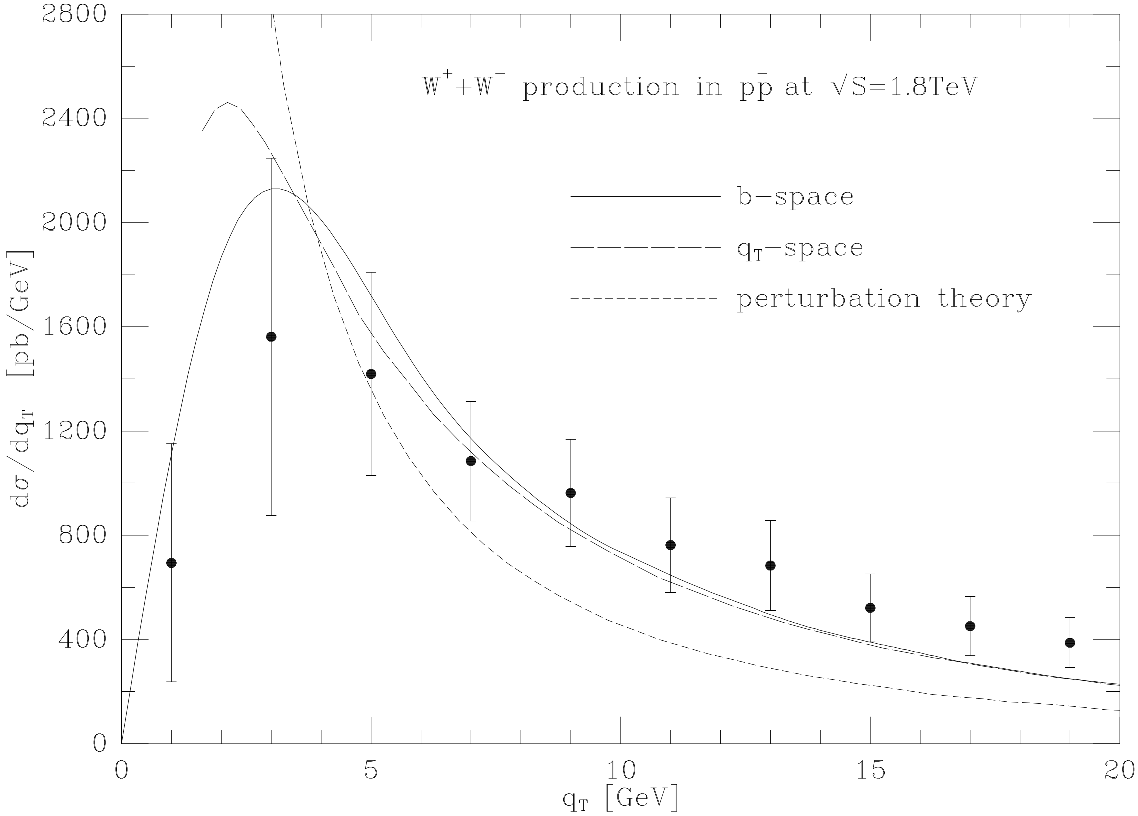

Figure 5: Comparison of various theoretical predictions for $W^+ + W^-$ $d\sigma/dq_{\rm T}$ with CDF data [23]. The b-space results were obtained with an effective gaussian form of F^{NP} ($g = 3.0$ GeV$^2, b_{\rm lim} = 0.5$ GeV^{-1}). We assumed $BR(W \to e\nu) = 0.111$.

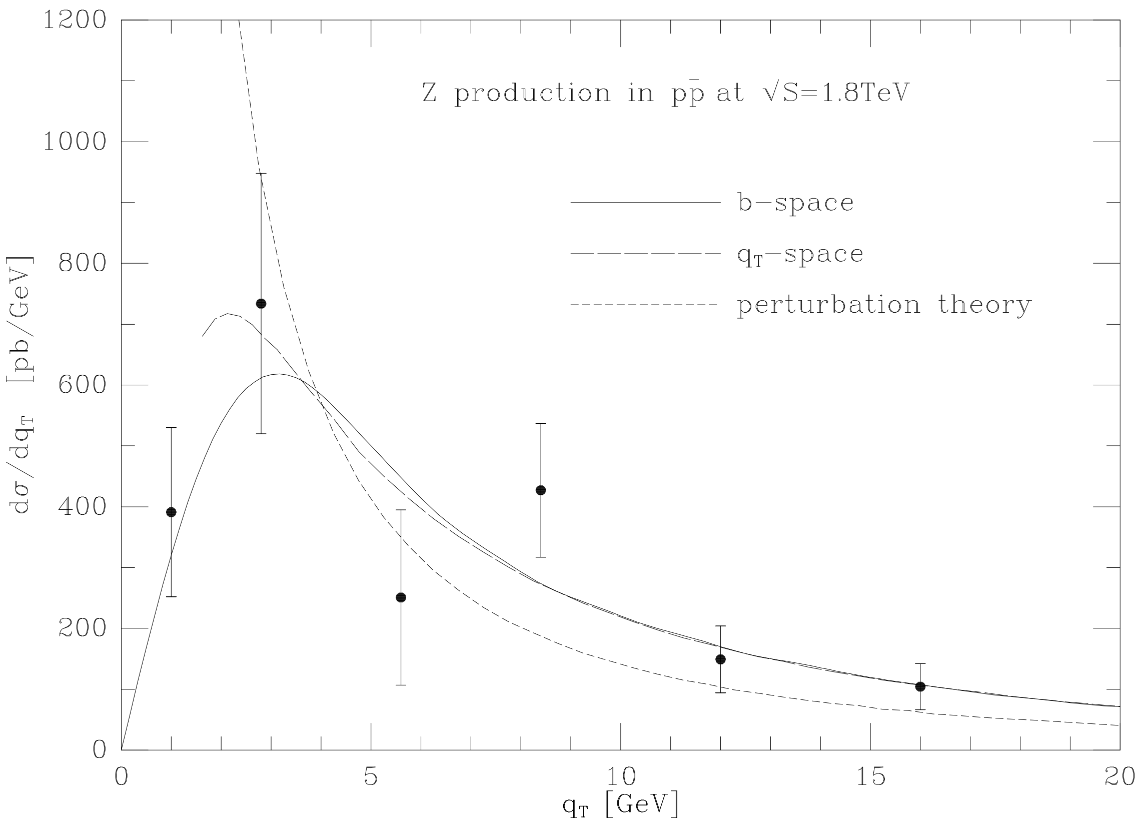

Figure 6: Comparison of various theoretical predictions for Z $d\sigma/dq_{\mathrm{T}}$ with CDF data [24]. The b-space results were obtained with an effective gaussian form of F^{NP} ($g = 3.0$ GeV$^2, b_{\,\mathrm{lim}} = 0.5$ GeV^{-1}). We assumed $BR(Z \to e^+e^-) = 0.033$.

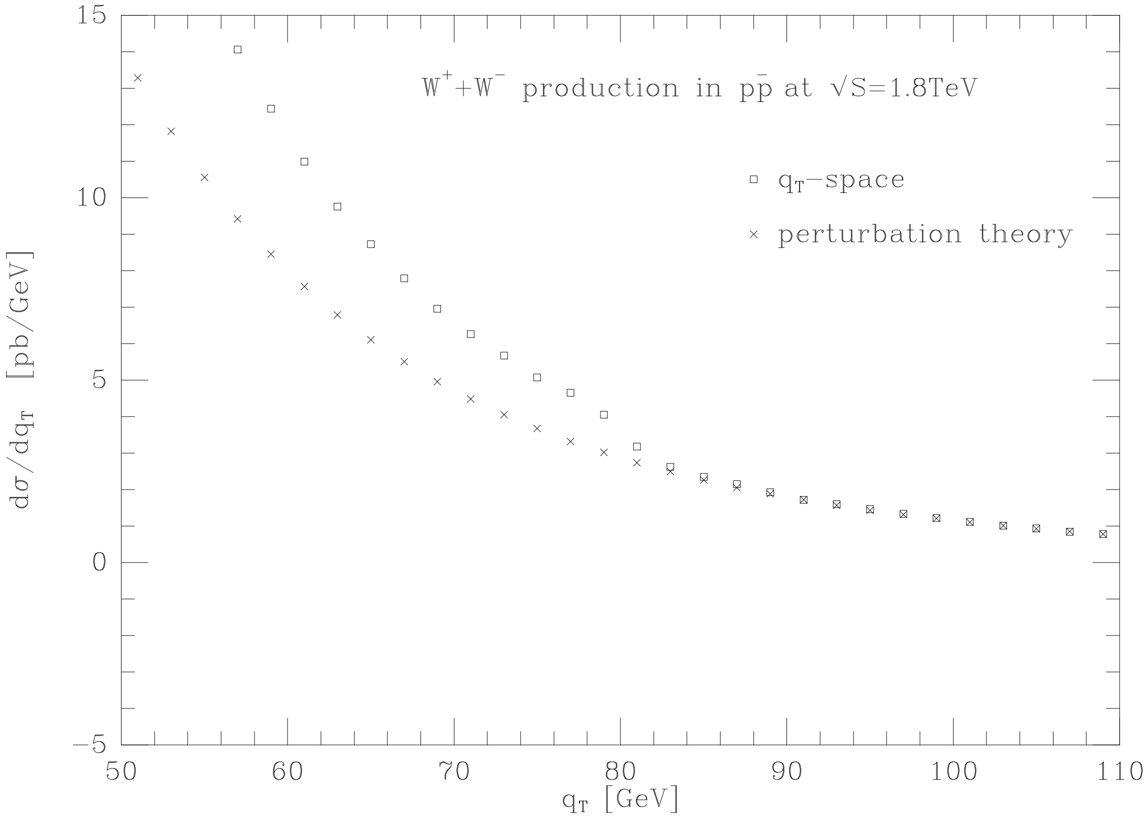

Figure 7: Comparison of the $q_{\rm T}$-space $d\sigma/dq_{\rm T}$ distribution for $W^+ + W^-$ production at $\sqrt{S} = 1.8$ TeV with $\mathcal{O}(\alpha_S)$ perturbative calculation. We assumed $BR(W \to e\nu) = 0.111$.

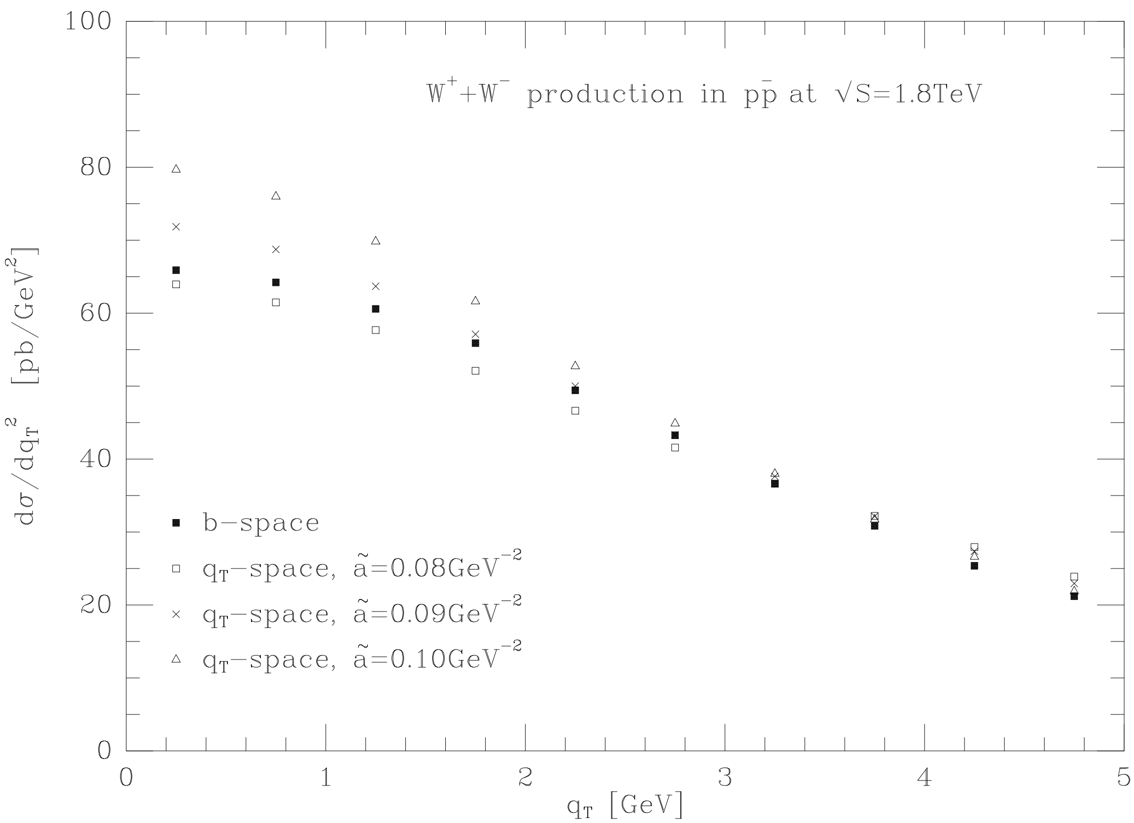

Figure 8: Various theoretical predictions for $d\sigma/dq_{\rm T}^2$ in $W^+ + W^-$ production at $\sqrt{S} =$ 1.8 TeV. The b-space results were obtained with an effective gaussian form of F^{NP} ($g =$ 3.0 GeV$^2, b_{\rm lim} = 0.5$ GeV^{-1}). The $q_{\rm T}$-space predictions correspond to $q_{\rm Tlim} = 4.0$ GeV.

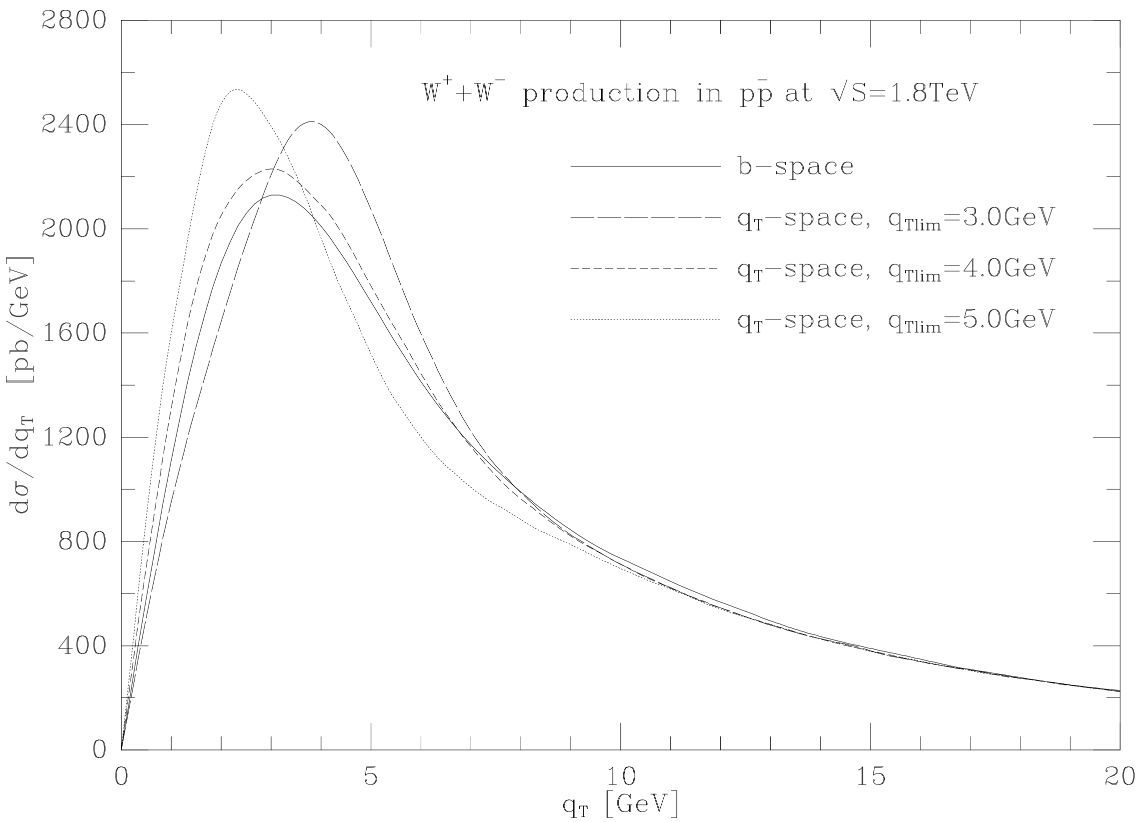

Figure 9: Various theoretical predictions for $d\sigma/dq_{\rm T}$ in $W^+ + W^-$ production at $\sqrt{S} =$ 1.8 TeV. The b-space results were obtained with an effective gaussian form of F^{NP} ($g =$ 3.0 GeV$^2, b_{\rm \,lim} = 0.5$ GeV^{-1}). The $q_{\rm T}$-space predictions correspond to $\tilde{a} = 0.10$ GeV^{-2}. We assumed $BR(W \to e\nu) = 0.111$.

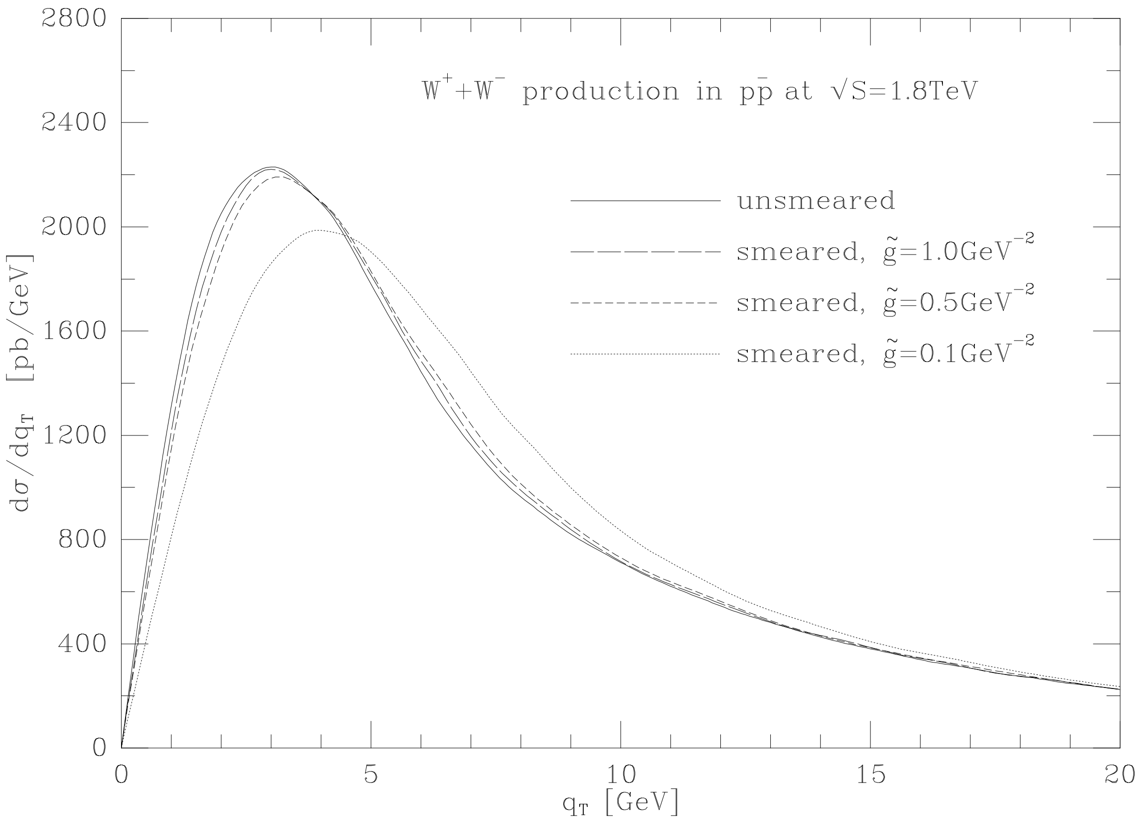

Figure 10: Effects of smearing in $q_{\rm T}$-space for $W^+ + W^-$ production at $\sqrt{S} = 1.8$ TeV. We used $\tilde{a} = 0.10$ GeV^{-2}, $q_{\rm Tlim} = 4.0$ GeV, and $BR(W \to e\nu) = 0.111$.